Sage Model Notes

GMSingleStageSquareWaveValves.scfn

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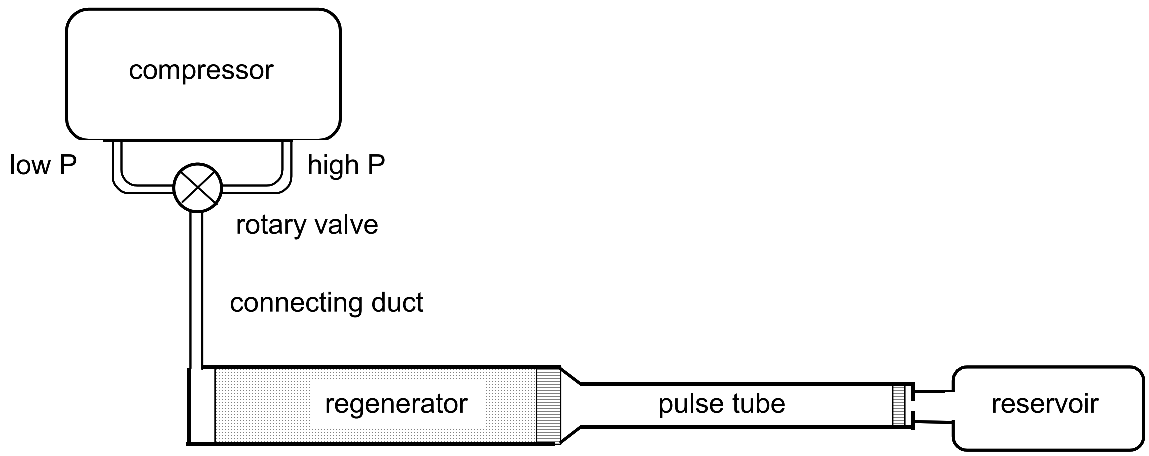
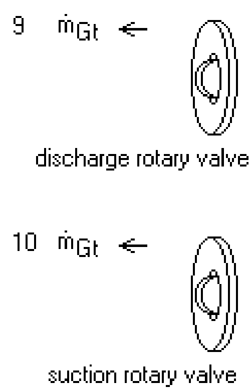
A variation of the GMSingleStage.scfn model with sinusoidal valve openings replaced by more abrupt square-wave openings. The model schematic remains the same:



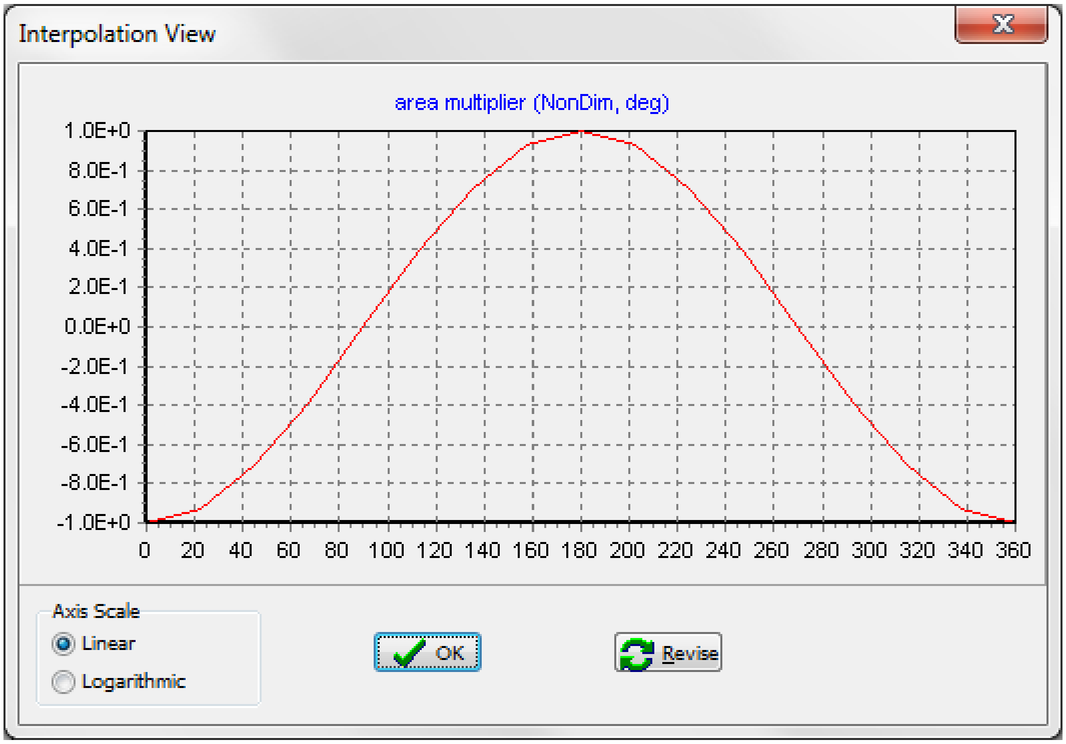
Compared to the original model GMSingleStage.scfn, the number of time nodes in the grid (NTnode) has been increased from 7 to 11 in order to resolve up to the 5th harmonic of the solution and the inputs for the time-dependent valves in the compressor submodel have been revised:



In the original model the flow restriction inputs were sinusoidal. For example, in the case of the suction rotary valve:

```
FRestrict     area multiplier (NonDim, deg)       0.0000E+00...
  ( 1.0000)E+00 Amp
  (-180.00)E+00 Arg
```

Viewed under the view interpolation option of the input dialog the open area multiplier as a function of time looked like this:

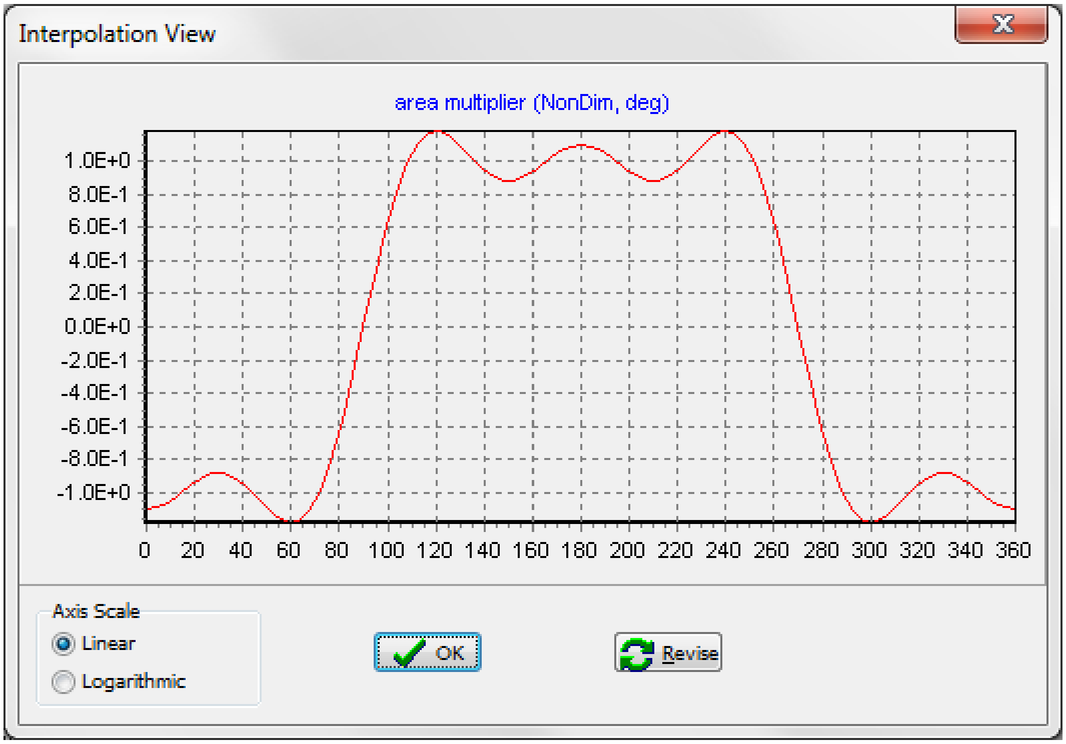


The actual open area is scaled by the above function only for values greater than 1.0E-3 (MinRestrict input), so in the original model the valve started to open around 90 degrees and closed again around 270 degrees with a sinusoidal variation between the two, to fully open at 180 degrees.

In the current model the flow restriction inputs are recast as square-wave functions, as represented by a 5-harmonic Fourier series. For example, in the case of the suction rotary valve the mean value, amplitudes and phases are:

```
 FRestrict = 2*OpenFrac - 1...
   (4*Sin(Pi*OpenFrac) / Pi,
    4*Sin(2*Pi*OpenFrac) / (2*Pi),
    4*Sin(3*Pi*OpenFrac) / (3*Pi),
    4*Sin(4*Pi*OpenFrac) / (4*Pi),
    4*Sin(5*Pi*OpenFrac) / (5*Pi)) Amp
   (-Phase,
    -2*Phase,
    -3*Phase,
    -4*Phase,
    -5*Phase) Arg
```

Which looks like this viewed under the view interpolation option of the recast variable dialog:



Now the valve opening is more abrupt, although a bit wiggly as a result of truncating the square-wave Fourier series at the 5th harmonic. Again, only the values above 1.0E-3 matter. So the valve abruptly opens at 90 degrees and abruptly closes at 270 degrees.

Variables OpenFrac and Phase in the recast expression are user defined inputs that determine the details of the square wave:

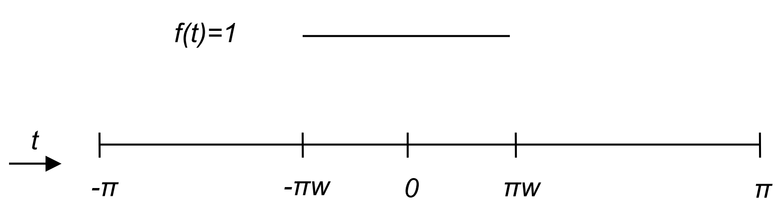
Inputs

OpenFrac	fraction cycle valve open (NonDim)	5.0000E-01
Phase	valve timing phase (deg)	1.8000E+02

OpenFrac defines the fraction of the cycle time the valve is open. Phase determines the valve timing — the time at the center of the open period. Changing Phase shifts the flow-restriction waveform one way or the other. In the discharge valve OpenFrac is the same but Phase = 0.

Math

The easiest step function to analyze is the defined in the interval $(-\pi, \pi)$ with the value $f(t) = 1$ for $-\pi w < t < \pi w$ and 0 otherwise.



w is a value between 0 and 1 corresponding to the OpenFrac input. The mean value is *w* and since it is an even function the Fourier series sine coefficients b_n are all zero. The cosine coefficients are:

$$a_n = \frac{1}{\pi} \int_{-\pi w}^{\pi w} \cos nt \; dt = \frac{1}{n\pi} \left[\sin(n\pi w) - \sin(-n\pi w) \right] = \frac{2}{n\pi} \sin(n\pi w)$$

For the case w = ½ (open half the time) this works out to $a_n = \frac{2}{n\pi}$ if *n* is odd, 0 if even.

In polar format, the amplitude coefficients are $c_n = a_n$ and phases $r_n = 0$ under the transformations:

$$c_n^2 = a_n^2 + b_n^2$$

$$\tan r_n = -\frac{b_n}{a_n}$$

To shift the center of the step interval to $t = \theta$, r_n is shifted by amounts $-n\theta$. See *Fourier Series* in the Sage User's Guide for more information.

To prevent the wiggles in the function rising above zero and interfering with the valve open time, the mean value in the Frestrict recast expression is reduced by 1 and the amplitudes are doubled. Thus, the square wave ranges from -1 to 1 in the Sage model.