### Sage Model Notes

### InLinePTR.scfn

D. Gedeon

12 April 2010 (revised 8 August 2024)

A model for a single-stage pulse-tube cooler with pulse-tube arranged in-line with regenerator. Similar to the CoAxPTR model without so many geometrical constraints. Here is a rough schematic of the physical layout:

piston				
compressor	connecting duct			
	regenerator	pulse tube	Inertance tube	reservoir

At the top-level the Sage model consists of a compressor submodel and a cold-head submodel with a connecting duct between the two:

🔝 In-Line PTR					
rstdy 4 P pressure source	4 $P_{stdy} \leftarrow \longrightarrow \dot{m}_{Gt}$ 5 5 $\dot{m}_{Gt} \leftarrow \longrightarrow \dot{m}_{Gt}$ 6 6 $\dot{m}_{Gt} \leftarrow \longrightarrow \dot{m}_{Gt}$ $\rightarrow \leftarrow \square \rightarrow \leftarrow \square$ $\rightarrow \frown$ $\rightarrow \leftarrow$ $\rightarrow \frown$ $\rightarrow \frown$ $\rightarrow \frown$ $\rightarrow \frown$ $\rightarrow \rightarrow$ $\rightarrow \frown$ $\rightarrow \frown$ $\rightarrow \frown$ $\rightarrow \leftarrow$ $\rightarrow \rightarrow$ $\rightarrow \frown$ $\rightarrow \frown$ $\rightarrow \frown$ $\rightarrow \rightarrow$ $\rightarrow \rightarrow$ $\rightarrow \frown$ $\rightarrow \frown$ $\rightarrow \frown$ $\rightarrow \rightarrow$ $\rightarrow \frown$ $\rightarrow \frown$ $\rightarrow \frown$ $\rightarrow \rightarrow$ $\rightarrow \rightarrow$ $\rightarrow$ $\rightarrow \rightarrow$ $\rightarrow \rightarrow$ $\rightarrow$ $\rightarrow \rightarrow$ $\rightarrow$ $\rightarrow$ $\rightarrow$ $\rightarrow$ $\rightarrow$ $\rightarrow$ $\rightarrow$				
<b>Compressor</b> The compressor consists of a single constrained piston driving a compression space:					



Within the *constrained piston and cylinder* component there is a constrained piston with an area attachment connected to the compression space. That area *A* is an independent input by default but in this case recast to 0.25\*Pi\*Sqr(Dshell) so it automatically adjusts to the piston diameter Dshell input of the *constrained piston and cylinder*.

### **Cold Head**

Within the cold head (*in-line head stage 1*) are a number of components that simulate a in-line pulse-tube arrangement. The reason for the naming '*stage 1*' is to anticipate the possibility of copying the entire submodel and pasting it back into the model to implement a higher stage.



The components are ordered in rows of decreasing temperature downward and horizontally in order of their physical layout. The *main rejector*, *acceptor* and *secondary rejector* components house copper heat-exchanger screens. The *regenerator* component houses a random-fiber regenerator. The *Ptube* component represents a tapered pulse tube while the *acceptor transition* and *sec rej transition* components represent minor transitional volumes. The *nrtube* and *reservoir* components model an intertance-tube phase shifter terminated by a fixed volume reservoir.

The *ambient parasitic source* anchors only parasitic wall thermal conductions. The *stage 1 source/sink* anchors wall conductions as well as cooling-power via conduction to the *acceptor* heat exchanger.

Were this part of a two-stage model there would be another positive facing  $\dot{m}_{Gt}$  connector created as a positive gas inlet within the gas sub-component of either the *acceptor* or *acceptor transition*. This connector would be promoted up to the root model level for connection to the *main rejector* of the second stage, thereby representing a pneumatic connection between the first and second stages.

There are a number of user-defined inputs and outputs at the submodel level:

Inputs		
ODregen	regenerator matrix OD (m)	2.000E-02
IDptube	pulse tube ID (m)	1.200E-02
IDnrtube	inertance tube ID (m)	3.500E-03
Outputs		
QrejColdhe	6.371E+01	
Qlift1	first stage net lift	3.509E+00

The following Lower-level components have their inputs recast in terms of the above inputs.

#### main rejector

Canister ID is recast to equal the regenerator matrix diameter ODregen. The *conductive surface* sub-component models radial heat flow through the wires. It has input D recast to half the canister radius (0.25 \* Din).

#### regenerator

Canister ID is recast to equal the regenerator matrix diameter ODregen. The outer wall of the regenerator canister represents the pressure wall which is important for modeling thermal conduction loss.

#### acceptor

Canister ID as well as conductive surface *D* are recast as for the *main rejector*. There is also a solid conduction path to the *stage 1 source* temperature. This conduction path (*wall distributed conductor*) simulates a copper cylinder wall thermally bonded to the screens. Its input *D* is recast to the wall thickness *Wcan* inherited from the *acceptor* canister.

#### acceptor transition

This is an optional component designed to model a possible transition volume between acceptor heat exchanger and pulse tube. It is a tapered tube with diameter as a function of position defined by cubic spline interpolation pairs. The diameter at the negative endpoint is recast to ODregen and at the positive endpoint to IDptube. It has an adiabatic wall so that all cooling heat transfer takes place in the *acceptor* heat exchanger.

In reality the flow in such a transition likely to be non-uniform, undeveloped, even separated. All of which lie beyond the scope of the Sage 1-D model to resolve. So don't expect extreme accuracy of this component. It is mainly present to account for the volume of the transition.

#### Ptube

This is another tapered tube component with diameters defined by cubic spline interpolation pairs. The diameter at the negative endpoint, mid-point and positive endpoint are all recast to IDptube. Recasting to separate diameters would also be possible to represent a tapered pulse tube.

Accurate modeling of this component requires that the incoming flow at either end be uniformly distributed without any high velocity jets or other flow separations.

#### secondary rejector

Canister ID is recast to IDptube. Conductive surface D is recast as for the main rejector.

#### sec rej transition

Another optional transition component, this time modeling a transition volume (possibly a conical flow diffuser) between secondary rejector and inertance tube. It is a tapered tube component with diameters defined by cubic spline interpolation pairs. The diameter at the negative endpoint is recast to IDptube and at the positive endpoint to IDnrtube. It has an isothermal wall surface with heat rejected to that surface monitored by user-defined output QsrTrans1.

#### nrtube

A single tube with ID recast to IDnrtube representing an intertance-tube phase shifter. Sort of like a ¼ wave open-ended organ pipe with the open end being the end connected to the *reservoir*.

## **Cold Temperature**

The file is currently set up for a cold-end temperature of 70K. To change cold-end temperature change the *T* input for the *stage 1 source/sink*, which establishes the acceptor screen temperature by solid conduction. Also, you might want to change Tinit at the positive end of the regenerator and negative end of the pulse-tube. These changes are not strictly necessary because the regenerator and pulse-tube temperatures are solved, with Tinit only providing initial values.

## **Bottom-Line Outputs**

Net cooling power is available in the *in-line head stage 1* submodel user-defined variable *Qlift1*. It derives from the net heat leaving the *stage 1 source/sink* component — the heat flow absorbed by the helium in the *acceptor*, less the conduction losses down the regenerator and pulse-tube canisters.

Compressor PV power input is available in user variable *Wpv* in the *compressor* submodel. There is no model of the motor driving the compressor piston so electrical power input is not available.

# Optimization

The model contains a rudimentary optimization specification. The objective is to maximize net cooling power *Qlift1*, subject to compressor PV power equal to 100 W (*Wpv* = 100). Optimized variables are compressor piston amplitude *Xamp*, regenerator matix *Porosity* and intertance tube *Length*.

Also optimized is the stiffness *K* of the spring attached to the compressor piston in order that the required force provided by the drive motor (*FF*) be in phase with the piston motion (*FF.Real* = 0).

This optimization is intended as a starting point for more serious design optimizations.