Sage Model Notes

TwoStageCooler.scfn

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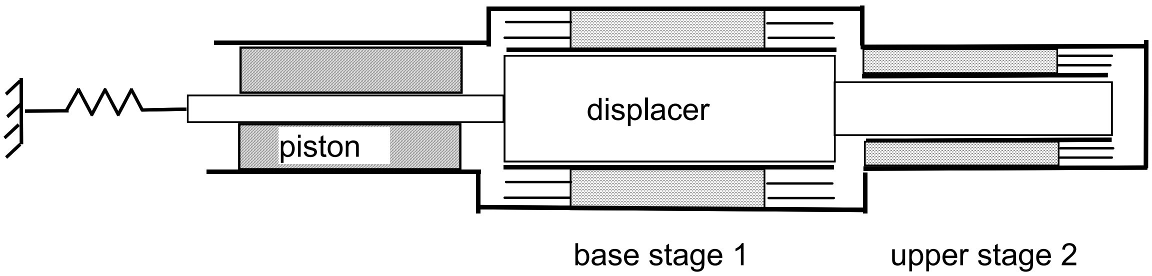
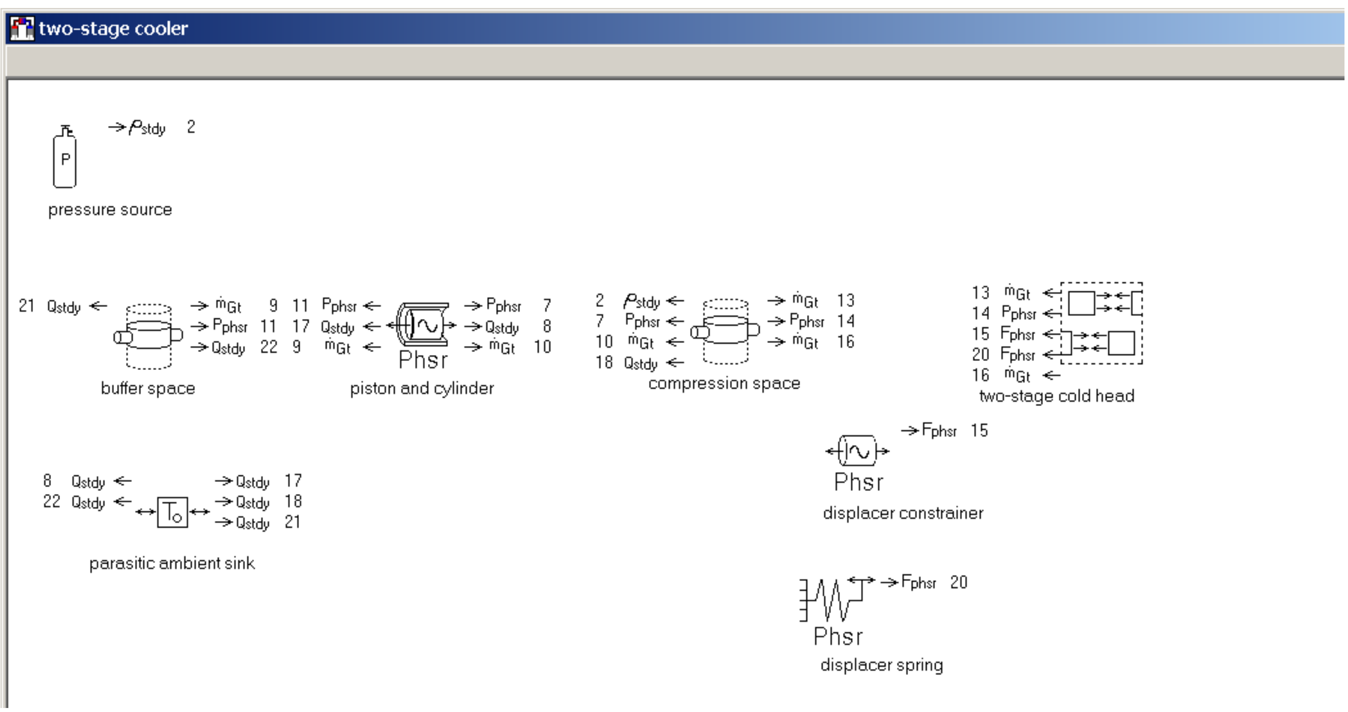
A model for a two-stage stirling-cycle cooler with an annular cold-head arrangement with heat exchangers and regenerators around a stepped thin-shell type displacer. Here is a rough schematic of the physical layout:

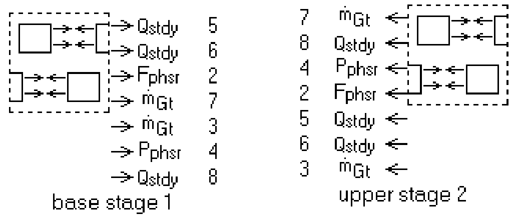


At the top level the Sage model consists of a piston driving a compression space directly connected to a cold-head submodel:

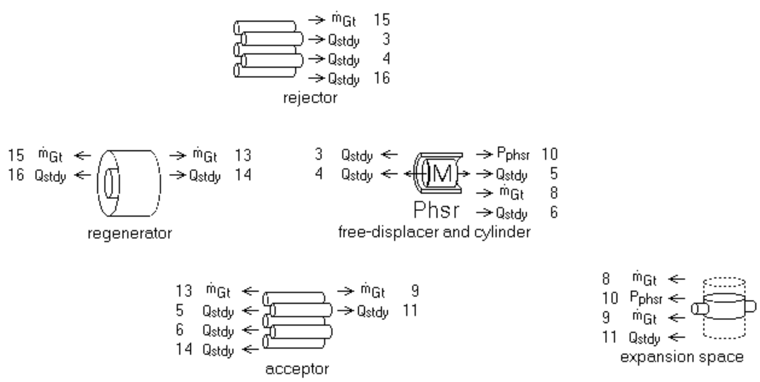


There is a clearance piston seal between the *buffer space* and the *compression space*.

Inside the cold head there are separate submodels for each stage:

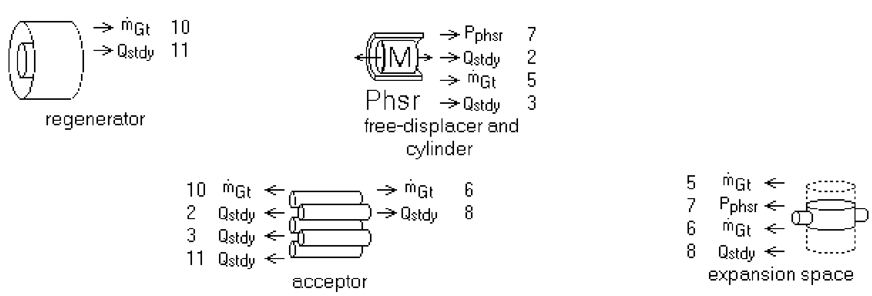


Within the *base stage 1* there are components representing the part of the displacer within and its cylinder, the annular regenerator and the two heat exchangers:



The components are arranged warm at top and cold at bottom. The *rejector* (warm heat exchanger) and *acceptor* (cold heat exchanger) are implemented as circular tubes which are configured to represent drilled holes in a annular copper blocks of conductive material. More on that below. Other heat exchanger types are possible. Sometimes heat exchangers are even omitted from *small* coolers. For example, if the heat absorbed by the acceptor is only a few Watts it may be possible to instead rely on the heat transfer in the space between the two stages. The *expansion space* is the variable volume space formed by the stepped displacer as it moves back and forth.

The *upper stage 2* submodel looks like this:



Unlike stage 1 there is no heat transfer at the warm (left) end of the *regenerator*. Instead the model relies on the *acceptor* and *expansion space* in stage 1 for that purpose. Otherwise is it identical to the base stage 1 submodel.

Recast Variables

There are a number of user-defined inputs in terms of which lower level inputs are recast in order to implement the geometric constraints of the physical hardware. These are:

Root Level

Drod	displacer rod diameter (m)	1.129E-02
Dpis	piston OD (m)	4.936E-02

Base stage 1

IDcyl	dis cylinder ID (m)	3.571E-02
Wcyl	dis cylinder wall thickness (m)	1.000E-03
IDPwall	regenerator pressure wall ID (m)	5.646E-02

Upper stage 2

IDcyl	dis cylinder ID (m)	1.241E-02
Wcyl	dis cylinder wall thickness (m)	1.000E-03
IDPwall	regenerator pressure wall ID (m)	2.259E-02

Most of the values shown are the result of optimization (see below).

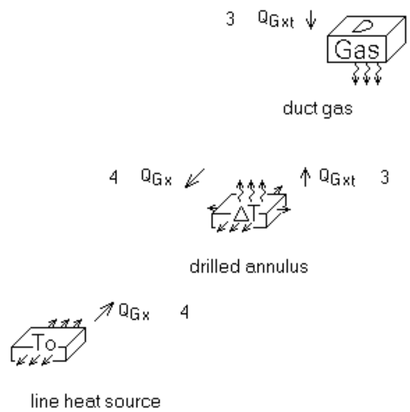
piston and cylinder
Within the root-level *piston and cylinder* the piston diameter Dshell is recast to the above *Dpis* and the frontal area attachments facing the compression and buffer spaces are recast to the piston frontal area less that of the displacer rod, consistent with the above schematic. However the piston leak only includes the leakage at the piston outer diameter and not any leakage through a displacer rod seal.

compression and expansion spaces
The wetted surface in the compression space is recast to four times the piston frontal area as a ballpark estimate of the correct value, good enough for initial design purposes. Likewise for the two expansion spaces except to four times the frontal area of the displacer within the particular stage.

regenerator
In each stage canister ID and OD are recast in terms of user-defined inputs of that particular stage and the thickness input defined in the regenerator.

free displacer and cylinders
Displacer diameters in the two stages are recast to fit inside their respective regenerator canisters and the cylinder wall thicknesses are recast to the respective stage-level *Wcyl* input. Their lengths are recast to equal the sum of regenerator plus heat exchanger lengths. The frontal areas are recast consistent with their diameters (and rod diameter in the case of the negative facing attachment in *base stage 1*). The displacers in the two stages are forced to move together with the same amplitude and phase by virtue of a force connection between them.

rejector and acceptors
All the heat exchangers look like this inside:



The intent of the model is to simulate an annulus of conductive material abutting the regenerator face with helium flowing through a number of uniformly-spaced axial holes are drilled. Recast variables morph the conductive surface (*drilled annulus*) to simulate the radial solid heat-flow path from an average hole (located mid annulus) to an isothermal source or sink at the outer wall (*line heat source*). Essentially the tube wall thicknesses are adjusted so the solid volume matches that of a drilled annulus and the transverse conduction length to match the radial annular thickness. The examples in the *How Do I Model* chapter of the Sage manual explain this sort of thing in more detail.

Temperatures

The file is currently set up for an ambient temperature of 300 K, a stage 1 temperature of 100 K and ultimate stage 2 temperature of 50 K.

To change ambient temperature change the *T* input for the *stage 1 parasitic ambient sink*, which establishes the temperatures of the buffer and compression space walls by solid conduction. The connections to the *piston and cylinder* component just anchor the shuttle heat transfer conduction path, which is moot because there is no temperature gradient. Also change the *Tinit* input for the *rejector* inside *base stage 1*. To change the cold temperatures change *Tinit* for the *acceptor* components inside the two stages. The heat exchangers employ isothermal *line source* components which inherit their fixed temperatures from their parent's *Tinit* input.

The various regenerators, variable-volume spaces and piston-cylinder components also have Tinit temperature inputs but these only establish initial values. They employ solid thermal conductors that adjust temperature with the solution and are anchored by endpoint thermal connections to the *drilled annulus* conductors within adjacent heat exchangers. Still, you might want to change the Tinit inputs for these components as well based on adjacent heat exchanger temperatures so as to produce a continuous temperature distribution initially. That will give a newly initialized model the best chance at convergence.

Bottom-Line Outputs

First and second stage net cooling powers are available root-level user-defined variable *Qlift1* and *Qlift2*. Included in these are the heat flows absorbed by the helium in the *acceptor* heat exchangers, less the conduction losses down the regenerator walls, displacer shells and their cylinder walls, including the shuttle heat transfer mechanism produced by the thermal interaction between shell and wall as they move relative to each other. regenerator and pulse-tube canisters.

Piston PV power input is available in root-level user variable *Wpv*. There is no model of the motor driving the piston so electrical power input is not available.

Free-Displacer Dynamic Analysis

At the root level the model includes a *displacer constrainer* component that drives the displacer at a specified amplitude and phase. The model is set up to help you design the appropriate displacer drive rod and spring to make the displacer run free at the same amplitude and phase of the *displacer constrainer*. This is done with Sage's optimizer by specifying the drive rod diameter *Drod* (root level user-defined input) and spring stiffness *K* (input of *displacer spring*) as optimized variables and implementing two constraints.

The constraints that make the displacer run free are, in English, that both components of the phasor force supplied by the *displacer constrainer* must be zero. These constraints are implemented within the *displacer constrainer* in terms of the forcing function *FF*:

Constraints
FF.Real = 0
FF.Imag = 0

After the optimization has converged the force connection between the *displacer constrainer* and *displacer* can be broken and the model should run in free-displacer mode at the same amplitude and phase.

Optimization

In addition to solving free-displacer dynamics the optimizer also optimizes a few other key inputs at the same time. The objective is to minimize PV power input *Wpv* subject to 100 W stage 1 cooling power (*Qlift1*) and 10 W stage 2 cooling power (*Qlift2*).

The main optimize variables are piston diameter Dpis, displacer constrainer phase angle *Xphase* and spring stiffness *K*. Optimized within the two stages are the key diameters IDcyl and IDPwall, the hole diameters *Dtube* for all heat exchangers, the regenerator *Length* and the regenerator matrix *Porosity*.

In the regenerators porosity is constrained to be greater than 0.70 because the correlations for heat transfer and flow resistance in random fiber regenerator matrices are not validated much below that. Porosity is also constrained to be less than 0.90 to avoid too flimsy a regenerator structure.

In the *compression space* and both *expansion spaces* the mean volume input (*Volume*) is optimized subject to the constraint:

FV.Mean = 1.5 * FV.Amp.1

Output *FV* is the Fourier-series representation of time varying volume and the constraint is designed to maintain a uniform buffer in the design against over-stroking and also to roughly include an allowance for the unswept volumes that occur in actual hardware.